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Risk control for staff planning in e-commerce warehouses

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Internet sale supply chains often need to fulfil quickly small orders for many customers. The resulting high demand and planning uncertainties pose new challenges for e-commerce warehouse operations. Here, we develop a decision support tool to assist managers in selecting appropriate risk policies and making staff planning decisions in uncertain conditions. Multistage stochastic modelling has been used to analyse risk optimisation approaches and expected value-based optimisation. Exhaustive numerical and practical validations have been performed to test the tool's applicability. We demonstrate, using a Dutch e-commerce warehouse, that the multi-period conditional value at risk appears to be most applicable.

Keywords: warehouse design; risk management; decision support systems; staff planning; e-commerce

1. Introduction

The opportunity for customers to order products online at any time with preferably short delivery times has resulted in increased planning uncertainties and fluctuating labour demand in supply chains. Here, we aim to study decision-making in uncertain conditions for scheduling and dispatching full-time staff and flexible and external work forces in Internet sales channels. Application of these concepts can be found in warehouses and other supply chain links in which shortage of labour might have a serious impact on essential performance indicators, such as response times and accuracy in delivery times. Typically, in e-commerce warehouses, large numbers of small orders have to be picked, packed and prepared for shipping in response to customer orders. Delays in order fulfilment may have negative consequences and the time windows for preparing orders for shipment may become limited (Gunasekaran, Patel, and Tirtiroglu 2001). On the other hand, over-staffing is not a solution due to the costs involved. Diverse working contracts, specific skills and tasks, and variations in employee productivity complicate staffing decisions (Fowler, Wirojanagud, and Gel 2008). Due to the high costs and risks involved, staffing decisions must be made as accurately as possible by incorporating all available information. We aim to derive a decision support tool that guides warehouse managers in their choice of risk control strategies in order to identify suitable staffing policies that match their goals.

Typically, staffing problems are examined for specific application areas due to the uniqueness and diversity of the underlying optimisation problems (Ernst et al. 2004). Many solution methods that take staff (or staffing) skills and types of work into account can be found in call center agencies (Aksin, Armony, and Mehrotra 2007) and the nursing sector (e.g. Jeang 1994; Eveborn, Flisberg, and Rönnqvist 2006). Staffing problems for warehouses in Internet sales channels differ in several ways from other application areas. First, the demand fluctuation accompanied by Internet retailing is high (Gong and De Koster 2011) due to the flexibility of customer options to shop independently of opening hours (Pechtl 2003). Additionally, distribution centres performing order fulfilment for other companies are often informed about their clients special promotions, advertisements, or discount offers on very short notice. Second, online purchases often result in product returns, as customers were unable to inspect the product prior to purchase. The number of returns and the labour effort involved in processing them creates additional uncertainty in the labour demand. Third, in most warehouse settings employees do not require specific skills to perform specific tasks. In contrast to other staff scheduling problems, the problem of scheduling the number of working hours for each employee to fulfil a specific workload remains problematic. The best possible scheduling for the upcoming planning period is essential. Accounting for stochastic influences becomes inevitable if the impact of even small failures is high. Any inaccuracy can lead to lost sales or unnecessarily high labour costs resulting from external personnel hired on short notice. Mismanagement can even lead to a loss of clientele or a less competitive position.

Stochastic models are used most often for staffing problems to deal with uncertainty in labour demands. Sadjadi et al. (2011) consider a period-wise staffing problem regulated by hiring and layoff policies. Their objective is to minimise all

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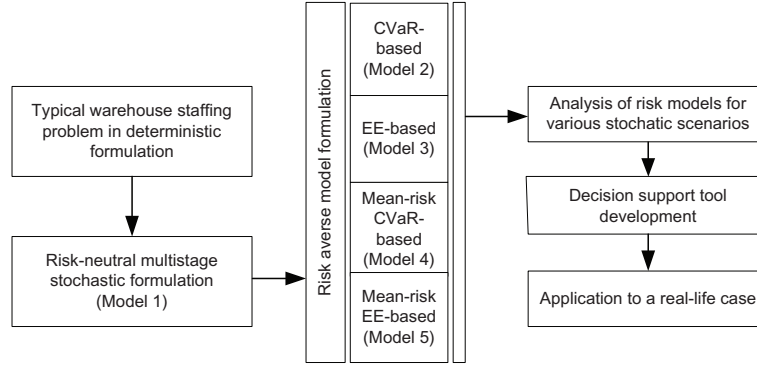


Figure 1. Research design.

related costs accompanied with hiring, layoff, labour shortage and surplus. [Bard, Morton, and Wang \(2007\)](#) derive a two-stage stochastic problem with recourse decisions. First-stage decisions concern full-time labour allocations and the amount of part-time labour. Second-stage decisions consist of the specific assignment of part-time employees to the schedule. Potential shortages are recovered with flexible labour. [Liao et al. \(2012\)](#) aim to derive a constant staffing level throughout the planning horizon and add uncertainty by using random mean arrival times. Similar to [Bard, Morton, and Wang \(2007\)](#), the authors highlight the necessity for sophisticated approaches for situations in which the system is very sensitive to data variation. Varying and unpredictable demand patterns in warehouses in Internet sales channels motivate even further exploration of risk aversion tools. The common approach of replacing the stochastic parameters with their deterministic expected values will lead to well-performing staffing policies only as long as the realisations of the stochastic parameters correspond approximately with expectations. Staffing policies that prepare the ‘mean case’ might fail to provide reasonable performances for the majority of possible scenarios. In their article on staff scheduling in mail processing ([Bard, Morton, and Wang 2007](#)) demonstrate the potential for 4% lower costs when recourse decisions are allowed during the planning horizon compared with the outcomes of the problem that solely optimises expectations of the entire time horizon. Multistage stochastic models are more sophisticated in tackling the uncertainties in staffing problems.

We aim to extend traditional stochastic programming methods by analysing the potential of risk-averse optimisation strategies. Furthermore, we design a decision support tool that guides warehouse managers in their choice of risk control strategies in order to identify staffing policies matching their purposes. To develop the tool we examine five optimisation approaches dealing with high variability in warehouse staff planning. We study the behaviour of various risk models, often used in financial risk management, in a representative warehouse situation for a variety of demand and shortage scenarios. The first model is a classical multistage stochastic programming approach that aims to minimise the expected total costs. Two other models utilise multistage risk measures, namely the multi-period conditional value at risk (CVaR) and the multi-period expected excess (EE), as the objective functions to be minimised. Lastly, we incorporate two mean-risk modelling approaches, each of which is based on one of the risk measures CVaR and EE.

The aim of this paper is to demonstrate for practitioners how stochastic optimisation models and financial risk measures can be used to incorporate and control risk in staff planning decisions. By means of a practical example we show how an appropriate application tool can be designed to make decisions.

Our research design is sketched in Figure 1. In Section 2, we specify our problem definition. In Sections 3 and 4, we explain the five models and subsequently introduce the set of warehouse scenarios in which the models are studied. The tool is designed in Section 5 by means of the insights obtained by performing numerical experiments for a warehouse staffing problem. In Section 6, we test the tool with the aid of a real-world case example of a Dutch commercial warehouse. Sections 7 and 8 discuss, respectively, conclusions and future research.

2. Problem definition

We consider the following representative e-commerce warehouse staffing problem as a basis for the modelling and analysis. A significant quantity of the daily work is performed by full-time employees with competitive salaries, working 40 h per week in varying shift schedules. The maximum number of working hours per day, as well as the minimum number of days off within the planning period are limited ([Eveborn and Rönnqvist 2004](#)). Second, there is more flexible labour available (i.e. part-time staff). For part-time staff we do not differentiate between single employees. A variable denotes the number of part-time staff workload hours. Third, the external workforce covers any remaining labour shortages. Especially during demand

Table 1. Parameter notations.

$\mathcal{T} = \{1, \dots, T\}$	Time horizon ($t \in \mathcal{T}$ is one day)
$\mathcal{A} = \{1, \dots, A\}$	Set of full-time workers
$c_a \in \mathbb{R}$	Costs of full-time worker per hour
$c_p \in \mathbb{R}$	Costs of part-time worker per hour
$c_e \in \mathbb{R}$	Costs of external worker per hour
$h_{\max} \in \mathbb{N}$	Maximum number of working hours for a full-time worker per day
$D_{\text{off}} \in \mathbb{N}$	Minimum number of days off of full-time workers in a period of length P
$\rho \in \mathbb{R}$	Maximum fraction of part-time relative to full-time workforce

Table 2. Decision variable notations.

$x_a^t \in \{0, 1\}$	Binary variable equals 1 if $a \in \mathcal{A}$ is working at day $t \in \mathcal{T}$, otherwise 0
$h_a^t \in \mathbb{N}$	Number of hours that $a \in \mathcal{A}$ is scheduled for work at day $t \in \mathcal{T}$
$p^t \in \mathbb{R}$	Number of working hours performed by part-time workers at day $t \in \mathcal{T}$
$y^t \in \mathbb{R}$	Working hours performed by externals at day $t \in \mathcal{T}$

peaks, postponements or delays of order fulfilments are not possible and, especially in e-commerce contexts, not desired. Thus, in our problem description we assume that demand has to be fulfilled immediately. Warehouses often work together with temporary employment agencies. As such, they can request labour on short notice in order to fulfil exceptionally high or unexpected demands due to, for example, seasonal peaks or discount sales. Recourse decisions to hire externals might have different reasons, namely to avoid more expensive over-time of full- and part-time staff, or postponement of work to the next shift. The latter can cause expensive delays in the order fulfilment. Similar to [Bard, Morton, and Wang \(2007\)](#), recourse costs exceed the regular working hour costs so there is an incentive to find schedules with little use of externals, little overtime or fewer delays. We assume that all employees are equally productive and that there is only one type of work (i.e. department). This is justified by the fact that the different computer-supported tasks (e.g. order picking, sorting, packaging and inspection) usually are easily learned and performed by all employees. The real-life case presented in Section 6 confirms this assumption. Lastly, we restrict the staffing problem using the commonly used condition that the ratio of part-time to full-time labour is limited ([Bard, Morton, and Wang 2007](#)). This is often used if flexible employees are accompanied by lower costs than full-time employees and a certain level of full-time staff assignment is desired.

2.1 Notations

To formulate the model let $\mathcal{T} = \{1, \dots, T\}$ denote the discrete time interval which describes the planning horizon. We assume a single shift mode, so that one $t \in \mathcal{T}$ represents, for example, one day. The warehouse manager can allocate full-time employees $\mathcal{A} = \{1, \dots, A\}$ to specific days $t \in \mathcal{T}$ with the help of the binary variable x_a^t and a specific number of hours by choosing h_a^t for $a \in \mathcal{A}$ and $t \in \mathcal{T}$. As mentioned above full-time employees receive a constant salary denoted by the cost parameter c_a stating the costs per employee per hour. The total costs of full-time labour are independent of the shift schedule. This concept describes a basic labour level that is always available but limited by a maximum number of hours per day and a minimum periodic number of days off. The part-time labour at day $t \in \mathcal{T}$ is denoted by p^t and solely restricted by the ratio $\rho \in [0, 1]$ of workload relative to full-time workload. The part-time labour is paid per hour assigned, their costs are denoted by the cost parameter c_p per hour. Recourse costs are denoted by the cost parameter c_e and the corresponding total number of labour hours is described by the variable y^t . Table 1 summarises the notation of the parameters used. The decision variables are listed in Table 2.

2.2 Deterministic model

We introduce the formal model by developing the deterministic optimisation model in which the objective is to minimise the total labour costs along the planning horizon. For the sake of clarification we summarise the constant costs of full-time labour by $C_{\mathcal{A}} = c_a \cdot h_{\max} \cdot T \cdot A$. Further, labour demand and shortage are known parameters in the deterministic model so

that we add here the following notation to Table 1:

$$\begin{aligned} d^t &\in \mathbb{R} && \text{labour demand at day } t \\ s^t &\in [0, 1] && \text{fraction of absent labour} \end{aligned}$$

The corresponding optimisation problem which minimises the total labour cost can be stated as follows:

$$\min TC = \min C_{\mathcal{A}} + \sum_{t=1}^T (c_p p^t + c_e y^t) \quad (1)$$

subject to:

$$d^t - \left((1 - s^t) \left(\sum_{a=1}^A h_a^t \right) + p^t + y^t \right) \leq 0 \quad \forall t \in \mathcal{T} \quad (2)$$

$$h_a^t - (x_a^t h_{\max}) \leq 0 \quad \forall t \in \mathcal{T}, a \in \mathcal{A} \quad (3)$$

$$\sum_{t=1}^T x_a^t - T + D_{\text{off}} \leq 0 \quad \forall a \in \mathcal{A} \quad (4)$$

$$p^t - \rho \sum_{a=1}^A h_a^t \leq 0 \quad \forall t \in \mathcal{T} \quad (5)$$

The objective function in Equation (1) consists of three cost components: the labour cost for full-time employees, the labour cost for working hours amassed by part-time employees and the labour costs for external labour. Constraint (2) ensures that demand is fulfilled at each time step. Constraint (3) limits the number of working hours per full-time employee per day. Constraint (4) regulates the number of days off within planning horizon \mathcal{T} . Constraint (5) restricts the amount of work that can be fulfilled by part-time employees relative to full-time employees at each time bin (Bard, Morton, and Wang 2007). This upper bound is needed since a part-time workforce allows for more flexibility than a full-time workforce. Thus, with Constraint (5) we create in the mathematical model the necessity to make use of a full-time workforce in order to reflect a realistic practical situation. External labour cannot be restricted as such, since our approach relies on complete recourse modelling, meaning that for any realisation of the stochastic parameters there is at least one feasible, potentially high-cost recourse option.

This deterministic formulation assists in developing the multistage stochastic formulation, which we introduce in the following section as a basis for our decision support tool.

3. Stochastic optimisation models

We use five different multistage stochastic programming approaches suitable for dealing with uncertainties and known widely from their use in financial risk management and other application areas.

Demand and labour shortage are not known in advance in this case, but described by a bivariate stochastic process $\{d^t, s^t\}_{t \in \mathcal{T}}$ with the realisations $(d_t^t, s_t^t) \in L_t$ for $t \in \mathcal{T}$. $L_t \subset \mathbb{R}^2$ denoting the finite set of realisations at time t . Finiteness of those sets is an important assumption that must be made, because it enables us to represent the possible outcomes with a scenario tree and to solve the resulting optimisation models with traditional solvers.

As a benchmark we propose a classical risk-neutral multistage stochastic modelling approach minimising the expected total costs. Clearly, with risk aversion the expected total costs are higher than with this model, but the solutions also provide more protection, although increases in expected total costs are also impractical.

The second and third models are likewise multistage stochastic optimisation models. Respectively, the CVaR and the EE are both minimised. The CVaR and the EE are related and represent two well-suited risk measures. Both incorporate the probability of certain risks and the extent to which, in these cases, the expected costs exceed a specific limit. Due to their mathematical properties both risk measures are often used in financial risk management (e.g. Artzner et al. 1999; Acerbi and Tasche 2002; Rockafellar and Uryasev 2002).

The CVaR and the EE are also suited for mean-risk modelling as applied in the fourth and fifth model. As risk optimisation and expected cost minimisation may utilise limited information on the stochastic parameters or cost structures, mean risk approaches form an alternative which incorporates both (Heinze 2008).

We introduce a notation describing conditional probabilities of realisations. Let the period to time t represent the historical realisations given by $((d_{l_1}^1, s_{l_1}^1), (d_{l_2}^2, s_{l_2}^2), \dots, (d_{l_{t-1}}^{t-1}, s_{l_{t-1}}^{t-1})) \in L_1 \times L_2 \times \dots \times L_{t-1}$. Then, we denote the conditional

probability of the realisation $(d_t^t, s_t^t) \in L_t$ to occur at time t by $\pi_{l_t}^{l^{t-1}}$, where $l^{t-1} = (l_1, \dots, l_{t-1})$. In other words, $\pi_{l_t}^{l^{t-1}}$ is the probability of a realisation (d_t^t, s_t^t) given the information that until time t the realisations $(d_1^1, s_1^1), \dots, (d_{t-1}^{t-1}, s_{t-1}^{t-1})$ have been observed. π_{l_1} denotes the absolute probabilities of outcomes at the first time steps.

3.1 Expected value-based multistage stochastic model

The objective function of the risk-neutral multistage stochastic programming model is a nested expression of expected costs for each time period, while minimisation is targeted over the sum of all realised costs of past time periods and expected costs of future periods. Here, the amount of externals used in time step t are incorporated by the stochastic recourse costs C^t which is a function of stochastic demand and shortage. The first expected value-based model of problem 1 can then be stated as

$$\min M1 = \min(C_A + c_p p^1 + {}_1 E[C^1 + c_p p^2 + E_2[C^2 + c_p p^3 + E_3[C^3 + \dots + E_T[C^T] \dots]]) \quad (6)$$

$$\begin{aligned} &= C_A + c_p p^1 + \sum_{l_1 \in L_1} \pi_{l_1} \left(C_{l_1}^1 + c_p p^2 + \sum_{l_2 \in L_2} \pi_{l_2}^{l_1} \left(C_{l_2}^2 + c_p p^3 \right. \right. \\ &\quad \left. \left. + \sum_{l_3 \in L_3} \pi_{l_3}^{l_2} \left(C_{l_3}^3 + \dots + \sum_{l_T \in L_T} \pi_{l_T}^{l_T^{T-1}} C_{l_T}^T \right) \right) \right) \end{aligned} \quad (7)$$

subject to

$$C_{l_t}^t \geq 0 \quad \forall t \in \mathcal{T}, l_t \in L_t \quad (8)$$

$$C_{l_t}^t - c_e \left(d_{l_t}^t - p^t - (1 - s_{l_t}^t) \sum_{a \in \mathcal{A}} h_a^t \right) \geq 0 \quad \forall t \in \mathcal{T}, l_t \in L_t \quad (9)$$

$$h_a^t - (x_a^t h_{\max}^t) \leq 0 \quad \forall t \in \mathcal{T}, a \in \mathcal{A} \quad (10)$$

$$\sum_{t=1}^T x_a^t - T + D_{\text{off}} \leq 0 \quad \forall a \in \mathcal{A} \quad (11)$$

$$p^t - \rho \sum_{a=1}^A h_a^t \leq 0 \quad \forall t \in \mathcal{T} \quad (12)$$

Decision variables in this formulation are x_a^t and h_a^t for assigning full-time labour to the schedule and p^t for defining the amount of work to be performed by part-time staff. Potential shortages in labour causes the recourse costs C^t for $t \in \mathcal{T}$, modelled with the help of Constraints (8) and (9), so that the objective function in model 1 (Equation (7)) states the total costs of full-time and part-time staff together with the expected recourse costs of each time period $E_T[C^T]$. The latter formulation is the key to stochastic modelling approaches, since it provides a deterministic ILP formulation, provided the number of realisations is finite at each time step. Constraints (8) and (9) are used to model the recourse costs. In accordance with the deterministic framework, Constraints (10)–(12) guarantee the feasibility of the decisions $(x_1^t, \dots, x_A^t, h_1^t, \dots, h_A^t, p^t)$. In the following we refer to the problem (7) as model 1.

3.2 Risk-based models

When fluctuations in the stochastic outcomes are high, the expected value problem might lead to impractical solutions for real applications. If the standard deviation of outcomes is high, any decision policy that solely adjusts to mean values can result in high recourse costs. To protect against this risk, we propose the first two risk-averse modelling approaches, which are based on the minimisation of the risk measures' conditional value at risk (CVaR, model 2) and expected excess (EE, model 3).

For single-period models, the CVaR for a level $\alpha \in (0, 1]$ expresses the expected cost of unfavourable realisations, i.e. high recourse costs C^t . In this case, parameter α defines instead unfavourable realisations. In Section 4.2 we elaborate on the effects of different values for α . To define the CVaR for one period let Z be a random variable. The CVaR is then defined as follows:

$$\text{CVaR}_\alpha = \inf_{z \in \mathbb{R}} \left\{ z + \frac{1}{\alpha} E[(Z - z)^+] \right\},$$

where

$$(Z - z)^+ = \begin{cases} Z - z, & \text{if } Z - z \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

The CVaR for a level α expresses the expected cost, if one of the $\alpha \cdot 100\%$ worst realisations occur. We can transfer the CVaR into a multi-period risk measure of our recourse costs C^t . With $\alpha = (\alpha_1, \dots, \alpha_T) \in (0, 1]^T$ the CVaR $_{\alpha}$ -based formulation of our problem becomes (model 2)

$$\begin{aligned} \min M2 = \min C_{\mathcal{A}} + \sum_{t=1}^T z_t + c_p p^1 + \frac{1}{\alpha_1} E_1 \left[Z^1 + c_p p^2 + \frac{1}{\alpha_2} E_2 \left[Z^2 + c_p p^3 \right. \right. \\ \left. \left. + \frac{1}{\alpha_3} E_3 \left[Z^3 \dots + \frac{1}{\alpha_T} E_T[Z^T] \dots \right] \right] \right] \end{aligned} \quad (13)$$

$$\begin{aligned} = \min C_{\mathcal{A}} + \sum_{t=1}^T z_t + c_p p^1 + \frac{1}{\alpha_1} \sum_{l_1 \in L_1} \pi_{l_1} \left(Z_{l_1}^1 + c_p p^2 + \frac{1}{\alpha_2} \sum_{l_2 \in L_2} \pi_{l_2}^{l_1} \left(Z_{l_2}^2 \right. \right. \\ \left. \left. + c_p p^3 + \frac{1}{\alpha_3} \sum_{l_3 \in L_3} \pi_{l_3}^{l_2} \left(Z_{l_3}^3 + \dots + \frac{1}{\alpha_T} \sum_{l_T \in L_T} \pi_{l_T}^{l_{T-1}} Z_{l_T}^T \right) \dots \right) \right) \end{aligned} \quad (14)$$

subject to

$$Z_{l_t}^t \geq 0 \quad \forall t \in \mathcal{T}, l_t \in L_t \quad (15)$$

$$Z_{l_t}^t - (C_{l_t}^t - z_t) \geq 0 \quad \forall t \in \mathcal{T}, l_t \in L_t \quad (16)$$

and subject to the Constraints (8), (9) and (10)–(12).

Similar to the above definition the multi-period CVaR is modelled here with the additional variables z_1, \dots, z_T and the random variables Z_1, \dots, Z_T . The remaining risk model conditions correspond with the expected value-based formulation of model 1.

A related risk measure is the EE. Let $\beta \in \mathbb{R}_+$ be a predefined value and B again a random variable with values in \mathbb{R} . The EE for one period is formally defined by the expected value of the difference between β and all realisations of B greater than β , i.e.

$$EE_{\beta} = E[(B - \beta)^+]$$

Like the CVaR, the EE thus describes the risk of exceeding a specific limit. In contrast to the CVaR the EE defines this limit with a real number, independent of the number or probability of scenarios for which B exceeds β . Let $\beta = (\beta_1, \dots, \beta_T) \in \mathbb{R}$ the EE_{β} -based formulation of our problem can be stated as follows (Model 3):

$$\begin{aligned} \min M3 = \min C_{\mathcal{A}} + c_p p^1 + E_1[B^1 + c_p p^2 + E_2[B^2 + c_p p^3 \\ + E_3[B^3 \dots + E_T[B^T]]]] \end{aligned} \quad (17)$$

$$\begin{aligned} = \min C_{\mathcal{A}} + c_p p^1 + \sum_{l_1 \in L_1} \pi_{l_1} \left(B_{l_1}^1 + c_p p^2 + \sum_{l_2 \in L_2} \pi_{l_2}^{l_1} \left(B_{l_2}^2 + c_p p^3 \right. \right. \\ \left. \left. + \sum_{l_3 \in L_3} \pi_{l_3}^{l_2} \left(B_{l_3}^3 + \dots + \sum_{l_T \in L_T} \pi_{l_T}^{l_{T-1}} B_{l_T}^T \right) \dots \right) \right) \end{aligned} \quad (18)$$

subject to

$$B_{l_t}^t \geq 0 \quad \forall t \in \mathcal{T}, l_t \in L_t \quad (19)$$

$$B_{l_t}^t - (C_{l_t}^t - \beta_t) \geq 0 \quad \forall t \in \mathcal{T}, l_t \in L_t \quad (20)$$

and subject to the Constraints (8), (9), and (10)–(12).

As indicated above the main difference between the CVaR and the EE to optimise risks, is the definition of risk either by those realisations that belong to a specified percentage of unfavourable realisations (CVaR) or by those realisations that exceed a specific monetary limit (EE). Both formulations have advantages and disadvantages for specific settings on which we elaborate in Section 5. For example, the EE formulation might have no impact on risk aversion for high values of β_t , if the potential recourse costs are low. On the other hand, the CVaR formulation might suggest highly expensive over-staffing solutions for low values of α_t , especially if the standard deviations of the recourse costs are high. In this

case, model 2 would suggest staffing policies that prepare for relatively few expensive outcomes resulting in high total cost policies.

3.3 Mean-risk models

Next we combine the expected value problem in model 1 with a risk optimisation approach as in model 2 and 3. We consider mean-risk-modelling (Heinze 2008), which is the standard method for modelling a bi-criteria optimisation problem with the objective to minimise a certain risk expression in addition to the expected value of the objective function.

We begin with the mean-risk model which uses the CVaR (model 4). The CVaR-based mean-risk model at level $\alpha = (\alpha_1, \dots, \alpha_T)$ is given by

$$\begin{aligned} \min M4 = & \min C_A + \sum_{t=1}^T z_t + c_p p^1 + E_1 \left[C^1 + \frac{Z^1}{\alpha_1} + c_p p^2 + E_2 \left[C^2 + \frac{Z^2}{\alpha_2} + c_p p^3 \right. \right. \\ & \left. \left. + E_3 \left[\dots + E_T \left[C^T + \frac{Z^T}{\alpha_T} \right] \dots \right] \right] \right] \end{aligned} \quad (21)$$

$$\begin{aligned} = & \min C_A + \sum_{t=1}^T z_t + c_p p^1 + \sum_{l_1 \in L_1} \pi_{l_1} \left(C_{l_1}^1 + \frac{Z_{l_1}^1}{\alpha_1} + c_p p^2 + \sum_{l_2 \in L_2} \pi_{l_2}^{l_1} \left(C_{l_2}^2 \right. \right. \\ & \left. \left. + \frac{Z_{l_2}^2}{\alpha_2} + c_p p^3 + \sum_{l_3 \in L_3} \pi_{l_3}^{l_2} \left(C_{l_3}^3 + \frac{Z_{l_3}^3}{\alpha_3} + \dots + \sum_{l_T \in L_T} \pi_{l_T}^{l_{T-1}} \left(C_{l_T}^T + \frac{Z_{l_T}^T}{\alpha_T} \right) \dots \right) \right) \right) \end{aligned} \quad (22)$$

subject to the Constraints (8), (9), (15), (16) and (10)–(12).

In a similar manner it evolves the EE-based mean-risk model (model 5):

$$\begin{aligned} \min M5 = & \min C_A + c_p p^1 + E_1[C^1 + B^1 + c_p p^2 + E_2[C^2 + B^2 + c_p p^3 \\ & + E_3[\dots + E_T[C^T + B^T] \dots]]] \end{aligned} \quad (23)$$

$$\begin{aligned} = & \min C_A + c_p p^1 + \sum_{l_1 \in L_1} \pi_{l_1} \left(C_{l_1}^1 + B_{l_1}^1 + c_p p^2 + \sum_{l_2 \in L_2} \pi_{l_2}^{l_1} \left(C_{l_2}^2 + B_{l_2}^2 \right. \right. \\ & \left. \left. + c_p p^3 + \sum_{l_3 \in L_3} \pi_{l_3}^{l_2} \left(C_{l_3}^3 + B_{l_3}^3 + \dots + \sum_{l_T \in L_T} \pi_{l_T}^{l_{T-1}} \left(C_{l_T}^T + B_{l_T}^T \right) \dots \right) \right) \right) \end{aligned} \quad (24)$$

subject to the Constraints (8), (9), (19), (20) and (10)–(12).

Mean-risk approaches might be more applicable to situations in which either the expected recourse costs are so high that expected outcomes are being considered, or uncertainty is low such that a slight risk aversion combined with the minimisation of expected cost yields the best results.

4. Experimental design

In order to analyse the behaviour of the different models and to shape the decision support tool for staff-scheduling policies we solve all five optimisation models for various demand and shortage situations as represented in Section 4.1. In Section 4.2, we calibrate the risk measures to find suitable values for the scalars α_t and β_t , $t \in \mathcal{T}$. We will illustrate how different values for each of these control parameters affect the risk model solutions.

For all experiments we consider a setting with 20 full-time employees, who can work 5 out of $T = 7$ days (i.e. $D_{\text{off}} = 2$), with $h_{\text{max}} = 8$ h maximum per day. We set the costs of a full-time employee to 32€ per hour (based on Tsai, Liou, and Huang (2008)). Part-time labour can be employed with up to $\rho = 50\%$ of scheduled full-time labour at each day. The costs of part-time labour are 35€ per hour. Under-staffing by full- and part-time labour has to be recovered with external labour (overtime, delay costs, etc.), accompanied by costs of 50€ per labour hour. The number of full-time employees and the value of the control parameter ρ are determined by the specific warehouse situation. Here, those values have been selected to match the demand scenarios which we present in Section 4.1. Finally, a scaling factor is used in the tree approximation to create similar units in the bivariate stochastic process $\{d^t, s^t\}_{t \in \mathcal{T}}$. It is defined by the average demand divided by the average shortage and is used only during the scenario tree construction to ensure that both demand and shortage are equally affected by the approximation.

Table 3. Scenario sets.

Position 1 Demand mean	Position 2 Demand std	Position 3 Shortage mean	Position 4 Shortage std
130 (1)	5 (1)	0 (1)	0.01 (1)
150 (2)	10 (2)	0.01 (2)	0.02 (2)
170 (3)	20 (3)	0.02 (3)	0.03 (3)
	30 (4)	0.03 (4)	
	40 (5)	0.04 (5)	

We use a backward-scenario tree construction as an approximation procedure (Heitsch and Römisch 2009) to reduce the problem dimension of the deterministic programming model of a multistage stochastic model. In a pretest we derived from the approximation parameters (1) an initial tolerated gap between scenarios paths of $\varepsilon_{T+1} = 8$ and (2) a decrease rate of the tolerance $q = 0.8$. For more details on these preliminary experiments we refer to Tsai, Liou, and Huang (2008). Using this approximation all optimisation models could be solved in less than one minute.

4.1 Numerical data scenarios

An overview of the scenario that we studied is given in Table 3. For the sake of simplification, we introduce a notation based on the positions in Table 3 referring to a single scenario. For example, scenario ‘1231’ refers to the scenario with an average demand of 130 h, a standard deviation of 10 h and an average shortage of 0.02 with a standard deviation of 0.01.

Overall, we analyse how different characteristics, such as the average and predictability of demand and shortage, affect the solutions of the different models. The total number of different scenarios is 225.

4.2 Risk measure scaling

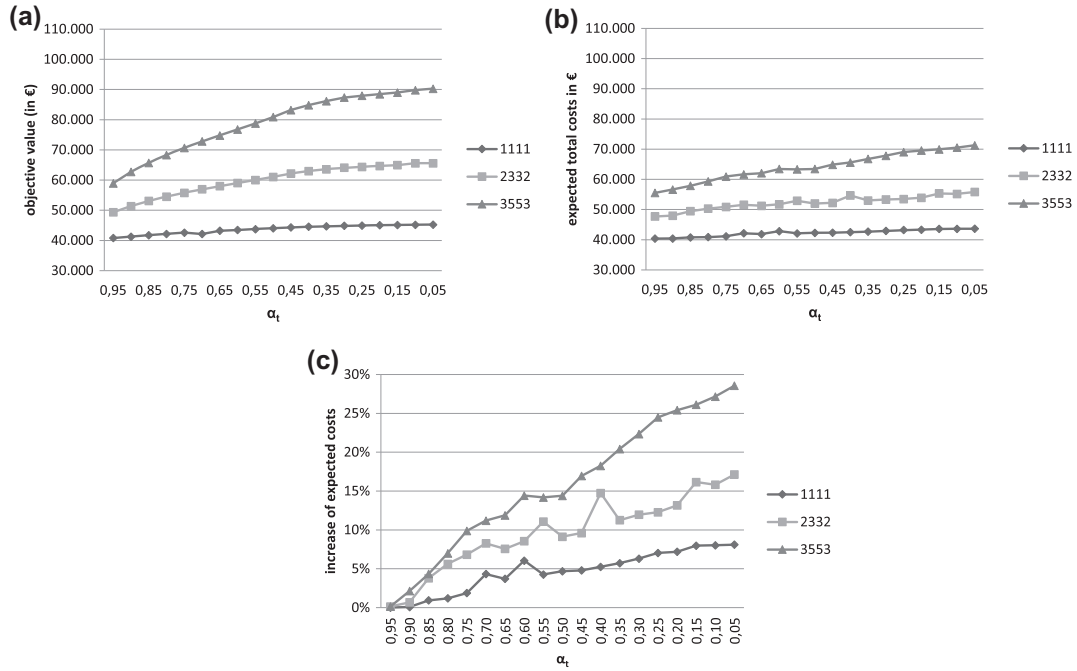
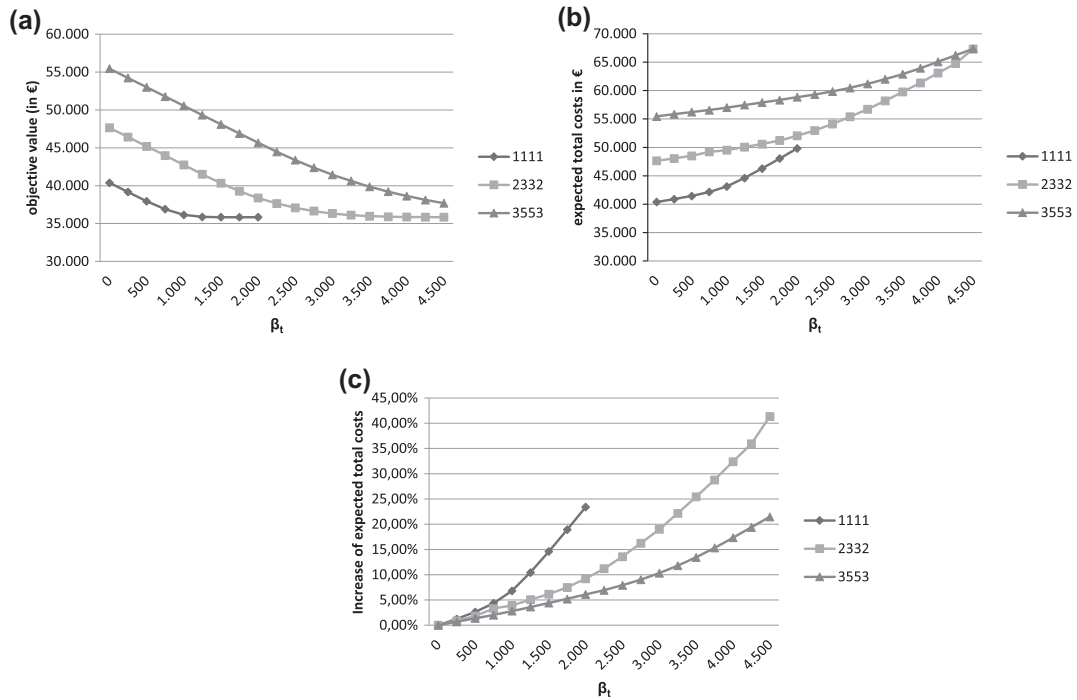
CVaR and EE are both risk measures that provide insights into the expected costs of negative outcomes. To clarify how different values of $\alpha_1, \dots, \alpha_T$ and β_1, \dots, β_T affect staffing policies we selected only a few scenarios, which we solved for model 1, as well as for the models 2 and 3 for various values of α_t and β_t for $t = 1, \dots, T$. In multi-period models differing values for different time steps are possible. However, such detailed experiments would not offer additional insights in the design of the decision support tool. Hence, in all experiments all values α_t and β_t were set, respectively, to equal values for all $t = 1, \dots, T$. We denote them by $\alpha = \alpha_1 = \dots = \alpha_T$ and $\beta = \beta_1 = \dots = \beta_T$. To illustrate the behaviour of the CVaR we depict the results for the solution of model 2 in Figure 2.

Model 2 minimises the expected costs for cases in which labour demand and shortage are realised in the $\alpha \cdot 100\%$ highest recourse costs. Figure 2(a) shows the expected costs of only the most expensive realisations. Obviously, $\alpha = 1$ results in an optimisation for the worst 100% of all realisations, and thus, in an optimisation of the expected total costs as in model 1. In Figure 2(b) we show the expected total costs per staffing policy.

The results show that for decreasing α both the objective value and the expected total costs increase. The objective value increases because the expected costs of a decreasing number of expensive realisations is considered. The expected total costs increase because the staffing policy adjusted to these outcomes results in over-staffing policies.

The results already indicate that model 2 is suitable for situations with lower potential recourse costs. For such scenarios as 1111 and 2332, the increase in costs appears to be lower than for scenarios with higher expected recourse costs, as seen in 3553 for example. More precisely, Figure 2(c) depicts the impact of risk protection on the expected total costs. While an $\alpha = 0.05$, i.e. a staffing policy suited for the 5% most expensive realisations can be achieved with 8% increased expected total costs for scenario 1111, an increase of 29% has to be expected for scenario 3553, where labour demand and shortage are high with high standard deviations. In general, it can be noted that, in particular, shortages in labour have a large impact on costs.

In Figure 3 we demonstrate the impact of using model 3. The EE denotes the expected costs above the barrier β of all scenarios in which they exceed β . In contrast to the CVaR, model 3 thereby defines risk by explicit costs. Figure 3(a) depicts the objective value of model 3. It decreases with increasing β since the objective function only measures the recourse costs that arise over and above β_t for each day. Higher β results in less additional costs even though the overall expected costs (depicted in Figure 3(b)) increase.

Figure 2. CVaR minimisation for various scenarios and α -levels.Figure 3. EE minimisation for various scenarios and β -levels.

For scenario 1111 this model cannot provide more risk aversion than with $\beta = 1500$ where the objective value reaches 35,800 € which are the constant costs of full-time employees. The corresponding expected total costs of this staffing solution are 46,300 € which is 15% above the expected total costs of the risk-neutral approach (model 1). In contrast to the CVaR model, model 3 shows higher increases for low-potential recourse cost situations than for potentially higher recourse costs.

Figure 3(c) shows the increase of expected total costs in more detail. With increasing β the scenarios 1111 and 2332 show particularly enormous cost increases. This is because only a few observations in these scenarios show potentially

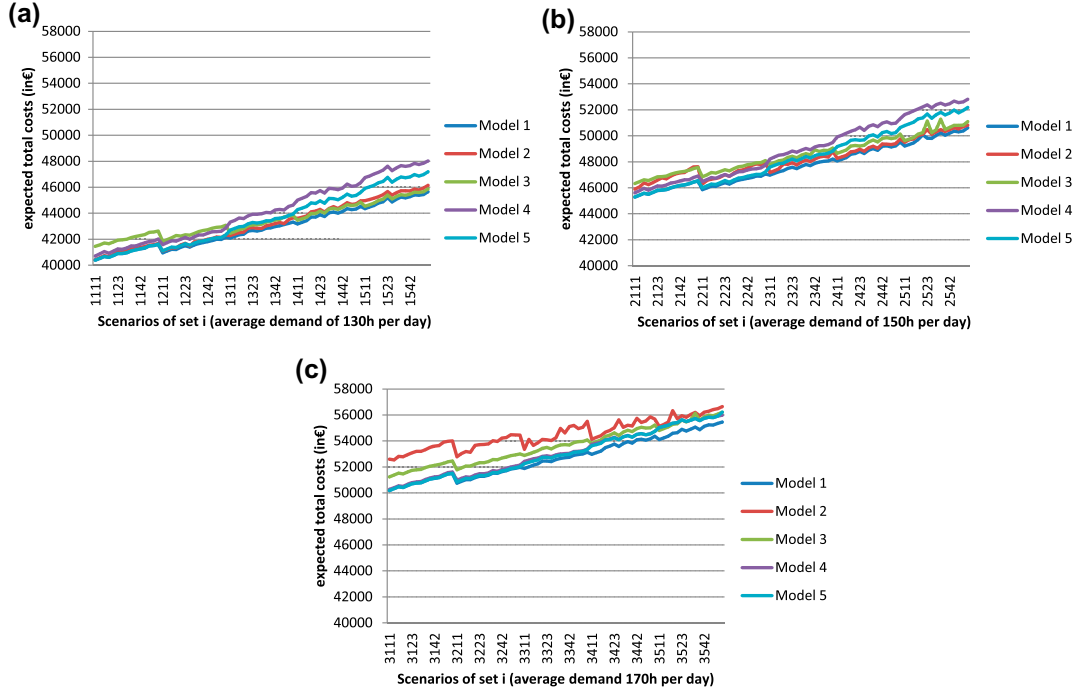


Figure 4. Results different scenarios.

high recourse costs (which exceed β) for which model 3 adjusts the staffing policy, resulting in unnecessary over-staffing. In contrast, in higher expected recourse cost scenarios expensive outcomes are also more likely. These are continuously considered in the optimisation for lower β , which results in an overall slower increase in expected total costs.

For the remaining experiments we proceed with the values of $\alpha = 0.9$ and $\beta = 500$. Namely, more risk aversion would result in expensive over-staffing policies and lower risk aversion produces a nearly risk-neutral optimisation for many of our scenarios.

5. Computational findings

In this section, we first discuss the results of solving the four data-sets with the five optimisation approaches to identify implications for a risk control tool. Section 5.2 presents our tool for choosing a suitable model for risk control in warehouse staff scheduling.

Obviously, model 1 will always provide the lowest expected cost solution, as these costs are directly minimised in model 1. As already seen in the previous experiments, the risk optimisation approaches sometimes differ greatly from this risk-neutral solution determined by model 1. Such extreme differences, however, imply high over-staffing solutions, which would be impractical to implement. On the other hand, we will also discover some scenarios in which no other model has an impact on the policy compared to model 1, i.e. no risk-aversion effect, and is therefore not suited for these scenarios. The aim of the following analysis is to identify those situations in which the risk and mean-risk approaches provide similar (i.e. slightly higher than) expected total costs to model 1 in order to derive a practical risk-aversion strategy.

5.1 Analysis of models

The results in Figure 4 are separated into scenarios with small, medium and high average labour demand.

The models show very different behaviour in the various scenarios. Generally, in line with previous literature (Wruck 2014), we can derive that high average labour shortage as well as uncertainties related to labour shortage significantly increase the expected costs for all five models. This effect can be observed also for scenarios with the same value for the average of labour shortage and solely increasing standard deviation. The same applies when the uncertainty of labour demand increases.

Model 2 provides reasonable expected total costs when the average labour demand and the potential recourse costs are low. The difference between the objectives in models 2 and 1 is smaller for scenarios with low-recourse costs than for situations with higher labour demand and subsequent higher recourse costs. With respect to CVaR-based risk measurement we observe that for low recourse costs scenarios model 2 provides lower costs than its mean-risk equivalent model 4. The mean-risk model 4, in turn, appears to be suited for situations with higher average labour demand and for lower standard deviations. The effect of risk optimisation disappears for model 4 when the standard deviation is too low (e.g. for scenario 3111 with a cost difference of 0.18% compared to model 1). In summary, we find that model 2 is suitable for situations in which low-recourse costs are expected (e.g. low demand relative to the available labour, low labour shortage). The mean-risk variant model 4 instead is a generally safe option for low uncertainty settings; it may have no effect when the potential recourse costs are high and variation is too low. In these cases, the risk measurement in the objective has minimal impact compared with the risk-neutral component (see Equation (21)).

The EE-based models 3 and 5 show nearly opposite behaviours. First, it appears that model 3 is not a good choice for low-potential recourse costs and low-variation scenarios. For the ‘easiest’ scenarios model 3 produces highly expensive solutions because they are almost deterministic. In these situations model 3 creates policies that prepare for cases where the daily recourse costs exceed 500€ (i.e. costs for external labour), which seldom occur. Model 3 becomes more applicable when demand uncertainty increases, since the number of scenarios with higher recourse costs also increases. In total, model 3 is suited for situations with higher recourse costs because the risk level is defined by a monetary value rather than by scenario probabilities. Model 5, in contrast, provides generally good results for small and medium variation, while the effect of risk protection might disappear for settings with uncertainties that are too low. In these cases the excess limit β is hardly reached so the results of model 5 almost coincide with those of model 1.

Finally, it appears that for high potential recourse costs and low uncertainty (i.e. standard deviation) both mean-risk models have lower costs than their risk-based alternatives. Because the risk aversion has little impact in these scenarios, the expected costs are similar to those yielded by model 1. On the other hand it is notable that when recourse costs decrease and uncertainty increases, the mean-risk models suggest more expensive staffing policies than their risk-based equivalents. This is somewhat surprising, as the expected total costs are also part of the minimisation objective in model 4 and 5. The reason for this is that for these cases too much attention is given to minimising the risk term in the objectives (21) and (23). The objective value reaches its minimum for higher expected total costs than a sole minimisation of the expected total cost would. Mean-risk models can therefore only be applied when the correct trade-off between the terms in the objective is given. They are ill-suited for situations with minimal uncertainties because the risk term has no impact and the solution is almost risk neutral. They are also not suitable for situations with extreme uncertainties because the risk term has too much impact and the solution becomes expensive. For such situations a scaling factor between the two terms in the objective could help to make a mean-risk model applicable. However, we do not consider such scaling factors here since they would shift the focus of the objective of the mean-risk model either towards the risk or towards the risk-neutral objective as our decision support tool would correspondingly suggest.

Our analysis showed that the choice of a risk-control approach in warehouse staff situations is mainly dependent on the level of uncertainty and the extent of the potential recourse costs of the staffing situation. Special uncertainty sources, such as peak days and labour effort through promotion-offers, fit in this concept and influence the outcome of risk optimisation mainly based on their impact on costs and their predictability. We therefore propose a risk approach determination based on these two factors and suggest the selection of a risk optimisation approach in accordance with the decision matrix depicted in Figure 5.

We do not specify precise limits for the quarters in the matrix, because these can result from more than one source in practical applications. In our experiments low uncertainty reflected very stable situations with a labour fluctuation of 5 h

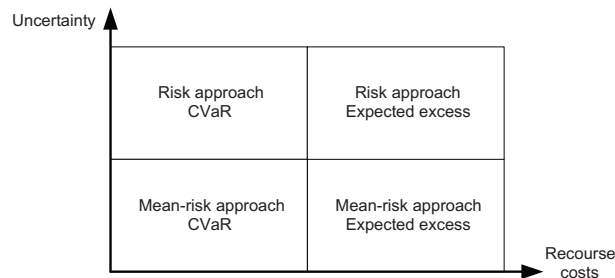


Figure 5. Decision matrix for risk optimisation approach.

and a shortage fluctuation of 1%. The highest uncertainty resulted from scenarios with 40 h of labour demand fluctuation and a shortage standard deviation of 3%. Moreover, the selection of an approach is not exclusive; applications with medium uncertainty and recourse cost situations might allow for two risk optimisation approaches with adjacent models in the matrix.

5.2 Decision support tool for risk management

We designed a decision support tool to identify an appropriate risk-measurement approach for a specific warehouse case. The tool also includes the specification of the warehouse situation in order to clarify the underlying optimisation model. Questions about uncertainties and their specific impact guided the decisions as to which model should be used and how to calibrate it for the specific purpose of the warehouse manager.

- Step 1 *Identification of the underlying optimisation problem*
Specify the desired planning horizon T . Specify potential fixed labour costs in the planning horizon (costs arising independently of specific labour schedules), set variable costs (i.e. labour costs per hour, which are paid only when workload is assigned), and other values and parameters related to the problem. Which constraints shape scheduling problem as shown for example in Section 2.2 (e.g. multiple shifts, shift lengths, and breaks)?
- Step 2 *Determine recourse options*
Which options are available for recovering short-notice labour shortage? Examples are overtime, employing external labour or postponement. Multiple options are possible; yet each shortage in labour demand must be recovered. The costs of each recourse option and potential limitations that must be added are specified with the help of constraints (e.g. overtime limits).
- Step 3 *Compose the resulting optimisation model*
Combine the scheduling problem of Step 1 with the recourse options of Step 2 into one single optimisation model as shown in model 1 in Section 3.1.
- Step 4 *Analyse historical data on labour demand and potential labour shortage*
If historical data show no significant trend in labour demand, especially trends suggesting that future observations differ considerably from past observations, a sufficiently large set of historical data can be used to determine labour schedules in the future.
If historical data shows respective trends then a suitable forecasting method incorporating those trends must be used to simulate future demands.
- Step 5 *Develop scenario trees*
Depending on the number of data observations, the complexity of the model (i.e. number of constraints) and the length of the planning horizon T , it might be necessary to reduce the number of observations with an approximation method.
- Step 6 *Analyse data*
Determine means and standard deviations of the uncertainty sources, such as labour demand, shortage, peak day intensities and promotion effort intensity.
- Step 7 *Decide on a risk optimisation approach*
Determine the most suitable approach by locating the scenario in the matrix in Figure 5. If no unique quarter can be chosen, proceed with the most suitable options in Step 8 and make a final decision based on experimental tests.
- Step 8 *Calibrate control parameters*
If a CVaR-based model has been selected in Step 7, an $\alpha_t \in (0, 1)$ has to be defined for each time step $t = 1, \dots, T$. High values of α_t create lower risk aversion, and thereby also lower total costs, than low values of α_t . Remember that $\alpha_t = 0.9$ realises an optimisation of the 90% highest recourse cost observations in the data-set.
If in Step 7 an EE-based model has been selected, choose a β_t for each $t = 1, \dots, T$. β_t has to be a monetary value and determines the excess limit. Remember that $\beta_t = 500 \text{ €}$ leads to an optimisation which minimises the average occurring costs on days were 500 € recourse costs are exceeded.
- Step 9 *Test the selection experimentally*
Solve the model(s) selected in Step 7 with the control parameters selected in Step 8. Compare the results with the expected total costs and analyse the resulting policy.
If the results do not match the desired risk aversion level go back to Step 8 and increase the risk aversion (lower α_t and higher β_t) until an acceptable solution is found.
If the results exceed the possible total costs go back to Step 8 and decrease the risk aversion (higher α_t and lower β_t) until an acceptable solution is found.

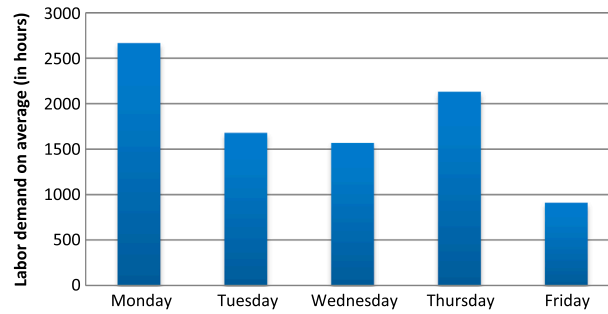


Figure 6. Average labour demand for single week days.

6. Case application

We apply the decision support tool in a study of the behaviour of the risk aversion approaches for the staffing problem of a Dutch commercial warehouse. In doing so, we can clarify how to determine the warehouse situation according to the decision matrix in Figure 5 and test the corresponding optimisation approach in contrast with the other approaches.

6.1 Warehouse setting

The warehouse handles order requests placed by individual consumers as well as business-to-business clients. Manual as well as semi-automated order picking was implemented. The warehouse has full-time and part-time employees, who work eight and four hours per day, respectively. Potential labour shortage is recovered with external labour via a temporary employment agency; postponement of customer requests is not possible and delays are avoided at all costs. Work is conducted in two shifts five days a week. The specific staff assignments in the early and late shift are independent from the decisions made on the number of labour hours to allocate for each day. Therefore we focus only on the number of full-time and part-time hours to be allocated.

The planning horizon of the company is one week. The costs of full- and part-time employees are 25 € per hour. External labour is paid with 19 € per hour. However, external employees are usually not familiar with the warehouse and the work required, so they are considered to be less productive with a productivity rate of 0.7 of the regular personnel employed by the warehouse. A cross-train policy is in place. Consequently, we do not distinguish between different types of work for different kinds of employees.

Labour demand is derived from incoming transactions for one year. The labour effort differs depending on whether an order consists of a single order line, a few order lines, or whether a request is fulfilled for a business partner and bulk orders have to be processed. We estimated the resulting demand of labour hours with a similar procedure as the company on the basis of the number of transactions per day. The average labour demand per day is 1785 h with a high standard deviation of 41%, but this average covers the entire year. The work load for specific weekdays, however, varies significantly, so the demand (except for predictable variation during the week days) shows a less uncertain behaviour if we consider it separately for each day of the week. The average labour demand throughout the week is depicted in Figure 6. The standard deviation for the single weekdays then constitutes 24% on average. No exact historical data on absenteeism of employees is available; however, it has been observed to be approximately 2% on average with a low standard deviation (1%).

6.2 Decision tool

Step 1 Identification of the underlying optimisation problem

The planning period of the company is one week and the warehouse is operational 5 days per week, resulting in $T = 5$ time steps. Staff is paid on an hourly basis. The staff members consist of full-time employees, who work eight hours when assigned to a shift and part-time employees who work four hours per shift. The costs of a labour hour are $c_w = 25$ € for full- and part-time staff members.

Step 2 Determine recourse options

Postponement or delays are not possible, so the only allowed recourse option is the recovery of labour shortage with external labour, with a productivity of 0.7 relative to both full-time and part-time staff and with costs of $c_e = 19$ € per labour hour.

Step 3 *Compose the resulting optimisation model*

We denote the number of full-time employees assigned at day t by x_{ft}^t and the number of part-time employees by x_{pt}^t . The risk-neutral model of this case has the following form:

$$\min c_w(x_{ft}^1 + x_{pt}^1) + E_1[C^1 + c_w(x_{ft}^2 + x_{pt}^2) + E_2[C^2 + \dots + E_T[C^T] \dots]]$$

subject to

$$\begin{aligned} C_{l_t}^t &\geq 0 & \forall t \in \mathcal{T}, l_t \in L_t \\ C_{l_t}^t &\geq \frac{c_e}{0.7} \left(d_{l_t}^t - p^t - (1 - s_{l_t}^t) \left(8x_{ft}^t + 4x_{pt}^t \right) \right) & \forall t \in \mathcal{T}, l_t \in L_t \end{aligned}$$

Step 4 *Analyse historical data on labour demand and potential labour shortage*

In the current practice historical data is used to predict future demand at this warehouse. Historical transaction data of one year is used, while the number of transactions is translated in labour hours similar to the warehouse's policy. Labour shortage is simulated according to the past observations with an average of 2% and a standard deviation of 1%.

Step 5 *Develop scenario trees*

The data did not require an approximation and could be used directly in the optimisation, since the time horizon was relatively short and the available data could be used.

Step 6 *Analyse uncertainty sources*

The warehouse situation described here shows a scenario with highly fluctuating labour demand in comparison with the previous experiments (approximately 24%). The potential recourse costs can be considered to be low, since the cost difference between pre-scheduling costs (25 € per labour hour) and recourse costs ($\frac{19}{0.7}$ € per labour hour) is low. In contrast to the warehouse example from the numerical experiments, in which part-time labour use (pre-scheduling costs) was also bounded and thereby high recourse costs occurred, here the warehouse could prepare for any outcome before the realisation, which naturally lowers recourse costs.

Step 7 *Decide on a risk optimisation approach*

Given the analysis of the case as described above our decision support tool suggests using a risk approach rather than a mean-risk model, and, given the low recourse costs, a CVaR approach instead of an EE-based model.

Step 8 and 9 *Calibrate control parameters and test the selection experimentally*

We summarise the last two steps by providing an overview of the outcomes that various values for α in the CVaR-based risk modelling approach as well as of the outcomes of various values for β in an EE-based risk approach. The total expected cost of the risk-neutral approach (i.e. model 1) are 49,900 €. We also include model 3 in this analysis since it could potentially be an option for this warehouse case and illustrates the expected total cost of models 2 and 3 by demonstrating the cost increase in comparison with model 1. Since a risk-averse decision-making can now be quantified, the final selection of the risk-aversion level (i.e. α and β , respectively) can be made by the decision-maker. Figure 7 shows the results of both risk models for a number of values of α and β , respectively.

6.3 Case results and implications

The decision made in this real-world warehouse situation was made by analysing the uncertainty sources and recourse costs in comparison with the numerical examples that we examined in the previous section. High labour demand fluctuations of approximately 24% motivated a focus on risk-based approaches rather than mean-risk models. They capture the uncertainty more clearly, and, therefore, allow for smarter risk-aversion policies than a combination of expected value and risk measure. Although not all models should have to be analysed in practical applications, we also solved models 4 and 5 for the same range of α and β values to illustrate the effect of this uncertainty on those models. We present the results in Figure 7. We find that both mean-risk modelling approaches have very low risk-aversion effects. The mean-risk model 4 (CVaR-based mean-risk) has a small effect for very low values of α , which means that this model could be used for optimising a combination of average outcomes and some expensive realisations. This concept however might be less intuitive than the corresponding CVaR-based risk approach. The EE-based mean-risk model, in contrast, shows a small risk-aversion effect only for small values of β , which implies that the uncertainties covered in the risk term are too minimal, so the trade-off between average costs and risks is imbalanced.

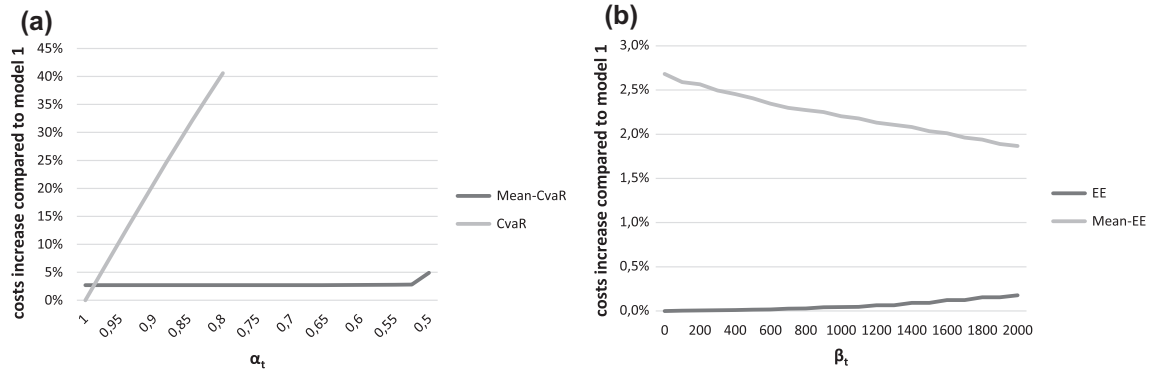


Figure 7. Cost increases with risk-based and mean-risk based modelling relative to risk-neutral optimisation.

The risk-based approach using the CVaR turns out to leave most flexibility in controlling risks in this real-life warehouse case. However, we also observe a strong sensitivity of the risk aversion on the expected total costs. Values for α which were already slightly smaller than one resulted in significantly increased costs. A value of $\alpha = 0.95$, for example, which suggests a staffing policy that is optimal for an average of the 95% most expensive scenarios, results in expected total costs that are 11% higher than a risk-neutral optimisation.

The warehouse should therefore use a CVaR-based modelling approach when aiming to take risk aversion into account in their staff planning method. However, the strong increase of the expected total costs with decreasing values of α also indicates that risk-aversion policies have the potential to become very expensive for this company. The reason for this behaviour is the minimal difference between pre-scheduling and recourse costs. In the risk-neutral approach the expected total costs are minimised which might lead to several days in the planning period in which recourse costs are accepted, because they result in the lowest costs. The CVaR model, in contrast, aims to minimise the risk of these recourse costs by over-staffing with slightly less expensive warehouse personnel. This results in a small cost advantage for busy days, but implies high unnecessary costs for the remaining days.

More generally we can conclude from these results that although the CVaR-based modelling approach is the one that is best suited for warehouse situations with high uncertainties, it becomes less practical when the recourse costs are only slightly higher than the pre-scheduling costs. Of course, in cases in which the pre-scheduling and recourse costs are nearly equal, risk-control policies are generally less important. Future research can consist of performing exhaustive numerical experiments with the aim to derive more structural results.

7. Conclusions

In this paper, we considered a staffing problem for e-commerce warehouses that is often accompanied by increased planning uncertainties. We analysed risk optimisation approaches in comparison with an expected value-based optimisation approach with the help of multistage stochastic modelling. We developed a decision support tool that can assist to control risks in specific practical warehouse settings. We applied the decision tool in a Dutch commercial warehouse case, in which risk control with the multi-period CVaR appeared to be the most applicable approach among the four available options.

The contribution of this work is twofold. First, we designed a decision support tool that can be used by practitioners together with an analysis and explanation of the decisions to be made and their impact on immediate outcomes. Testing the tool in a real-life case has demonstrated its applicability. For the case that we studied here the risk modelling approach with the CVaR has provided the best results. Furthermore, the paper might be used as a role model for practitioners to use stochastic modelling tools for staff planning.

Second, from an academic perspective we advance towards the use of stochastic modelling approaches in logistics problems, which has been recommended by many researchers (e.g. De Koster, Le-Duc, and Roodbergen 2007; Gong and De Koster 2011). Stochastic approaches allow for various kinds of risk management in an optimisation, several examples of which have been described here.

The use of risk measures in the context of logistics has also revealed a separate distinction compared from its application in financial mathematics. While uncertainty for financial products is usually accompanied with gains on the one hand and losses on the other, we observed a different effect. For example, increases in the variation of labour shortage lead to significantly

higher costs, even though the average of shortages is the same. Clearly, while the total costs suffer from highly expensive outcomes they do not profit from exceptionally low shortage realisations, leading to the observed effect. Risk control in logistics planning problems enables the decision-maker to quantify certain risks, enabling them to make more informed decisions about future planning.

8. Future research

In this section, we discuss several lines of future research. Stochastic modelling approaches always implicate a limitation regarding the problem size, since the models are solved by converting them into large-scale ILP formulations. Especially for longer time horizons the resulting optimisation problem becomes numerically complex. Future research might therefore focus on useful approximation methods for larger problems. Larger problems could, for example, result from the fact that online retailers (e.g. Amazon) have the tendency to add product categories and consequently experience a growth in their assortment. Next to that, cross-chain control centres are being established that handle the online sales for different retailers at the same time, which clearly increases the complexity and problem size of warehousing processes. Also the extension of the approach by other sources of uncertainty (e.g. availability of labour supply, call-back campaigns or unexpected promotion activities for products), by other recourse options (e.g. delivery of demand from another warehouse location or outsourcing operations), or by other applicable risk measures (e.g. via service level agreements with customers) could provide interesting insights. Furthermore, a refinement of the decision support tool can make it useful to other industries or a wider range of applications in the logistics sector. For example, applying the model for integral staff planning decisions over a group of warehouses. Specific questions to address are what additional parameters need to be considered? how to deal at the same time with different risk measures for each of warehouses hiring from the same pool of staff. Based on those kinds of factors it can be studied how to change the model and tool.

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